Chameli Devi Group Of Institutions

FDP on

Multi-Criteria Decision Making -MCDM Approach

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Introduction

Zeleny (1982) opens his book “Multiple Criteria Decision Making” with a statement:

“It has become more and more difficult to see the world around us in a unidimensional way and to use only a single criterion when judging what we see”

Introduction

- MCDM is a situation where an user want to take decision but he/she trapped under several factors & cannot be able to find the best alternative among the available ones.

- MCDM consists of two related paradigms:

  Multiple Attribute Decision Making (MADM).

  Multiple Objective Decision Making (MODM).

- MADM problems are assumed to have a predetermined, limited number of decision alternatives.

Introduction

- In MODM unlike MADM; the decision alternatives are not given. Instead the set of decision alternatives are explicitly defined by constraints using multiple objective programming.

- AHP(Analytical Hierarchy Process) is a powerful technique for such type of decision making.

Different decision making algorithm

- Multi attribute utility theory(MAUT)
- Cost-benefit analysis (CBA)
- Conjunctive and disjunctive methods
- Analytical Hierarchy Process (AHP)

MAUT

- Multi-Attribute Utility Theory (MAUT) is a structured methodology designed to handle the tradeoffs among multiple objectives.
- One of the first applications of MAUT involved a study of alternative locations for a new airport in Mexico City in the early 1970s.
- The end result is a rank ordered evaluation of alternatives that reflects the decision makers' preferences. The basis of MAUT is the use of utility functions.

Cost-benefit analysis (CBA)

- CBA is a systematic process for calculating and comparing benefits and costs of a project, decision .

  CBA has two purposes:

- To determine if it is a sound investment/decision.

- To provide a basis for comparing projects. It involves comparing the total expected cost of each option against the total expected benefits, to see whether the benefits outweigh the costs, and by how much.

Conjunctive and disjunctive methods

- These methods require satisfactory rather than best performance in each criterion.
- The conjunctive method requires that an alternative must meet a minimal performance threshold for all criteria.
- The disjunctive method requires that the alternative should exceed the given threshold for at least one criterion.

Analytical Hierarchy Process (AHP)

- AHP is a decision making technique for solving complex decision.
- It was developed by Thomas L. Saaty in the early 1970's.
- AHP helps decision makers find one that best suits their goal and their understanding of the problem.
- Users of the AHP first decompose their decision problem into a hierarchy of sub-problems,.

AHP-A method for MCDM

- Each of which can be analyzed independently. The elements of the hierarchy can relate to any aspect of the decision problem.

- Once the hierarchy is built, the decision makers systematically evaluate its various elements by comparing them.

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- The AHP employs an underlying scale with values from 1 to 9 to rate the relative preferences for two items.

Verbal Judgment of Preference	Numerical Rating
Extremely preferred	9
Very strongly to extremely	8
Very strongly preferred	7
Strongly to very strongly	6
Strongly preferred	5
Moderately to strongly	4
Moderately preferred	3
Equally to moderately	2

Methodology of AHP

- Identify the alternatives.
- Identify the criteria.
- Construct a pair wise comparison matrix for the criteria.
- Now, determine weights of all the criteria. For determining weights, first normalize the pair wise matrix by dividing entries in column by the sum of the elements in the corresponding column.

$$w_i = \frac{\sum_{j=1}^{n} (b_{ij})}{n}, \quad \text{for } i=1,2....n$$

Methodology of AHP

- Then find n different comparison matrices for alternatives (on the basis of each criterion).

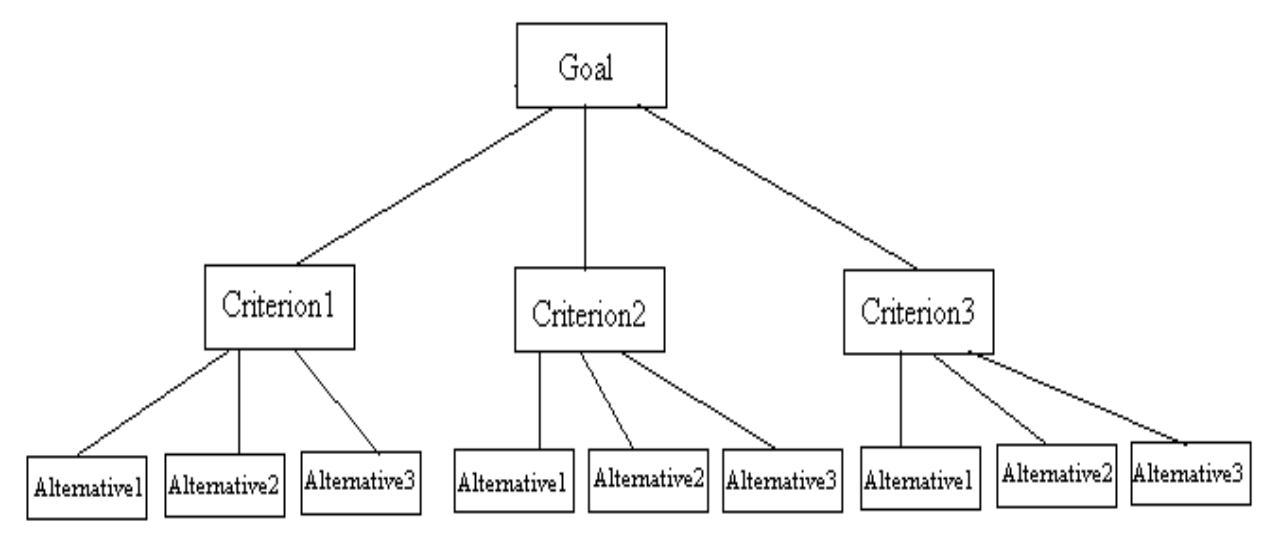
- Then normalize all these n matrices and calculate the weight corresponding to each matrix.

$$V_{ij} = \frac{\sum_{j=1}^{n} (b_{ij})}{n}, \quad \text{for } i=1,2....n$$

- Obtain the final results using these matrices and the weights. Whichever alternative has highest score, will be the most suitable choice to the user.

$$A[i] = \sum_{j=1}^{n} V_{ij} \times w_j$$

Structural Diagram of AHP



Example of AHP:OS Selection

Analysis of different Operating systems using AHP:

AHP assigns 1 to all diagonal elements & aij= 1/aji.

Suppose the comparison matrix for the criteria is,

	Cost	Security
Cost	$a_{11} = 1$	$a_{12} = 0.5$
Security	$a_{21} = 2$	$a_{22} = 1$

Here a21 = 2 indicates that Security is twice prior as compared to cost.

Now, as discussed earlier, normalized form of this comparison matrix will be,

Example of AHP

	Cost	Security
Cost	0.333	0.333
Security	0.667	0.667

- Weights w_1 and w_2 are also calculated here, which are calculated as per the formula described earlier

	average weights
w_1 =	0.333
w_2 =	0.667

Example of AHP

Now, let comparison matrices for three alternatives on the basis of each criterion are,

Cost	Windows	Linux	OS400
Windows	1	1	0.333
Linux	1	1	0.5
OS400	3	2	1

Comparison matrix for alternatives on the basis of cost

In this matrix, OS/400→ Windows has an entry 3, which indicates that OS/400 is three times costly than Windows.

Example of AHP

Security	Windows	Linux	OS400
Windows	1	1	0.5
Linux	1	1	0.5
OS400	2	2	1

Comparison matrix for alternatives on the basis of security

In this matrix Windows→ OS/400 has an entry 0.5, which indicates that Windows is half secure as compared to OS/400.

Now, after normalizing these two matrices, we will get,

Example of AHP

Cost	Windows	Linux	OS400
Windows	0.2	0.25	0.182
Linux	0.2	0.25	0.273
OS400	0.6	0.5	0.545

Normalized matrix for alternatives on the basis of cost

Security	Windows	Linux	OS400
Windows	0.25	0.25	0.25
Linux	0.25	0.25	0.25
OS400	0.5	0.5	0.5

Normalized matrix for alternatives on the basis of security

Example of AHP

- From these normalized matrices vectors S_1 and S_2 will be calculated

$$S_1 = \begin{bmatrix} 0.211 \\ 0.241 \\ 0.548 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.25 \\ 0.25 \\ 0.5 \end{bmatrix}$$

- Using S_1, S_2 and weights w_1, w_2, we will finally calculate final scores for the alternatives,

A [1] = (0.211 x 0.333) + (0.25 x 0.667)

A [1] = **0.237**

A [2] = (0.241 x 0.333) + (0.25 x 0.667)

A [2] = **0.247**

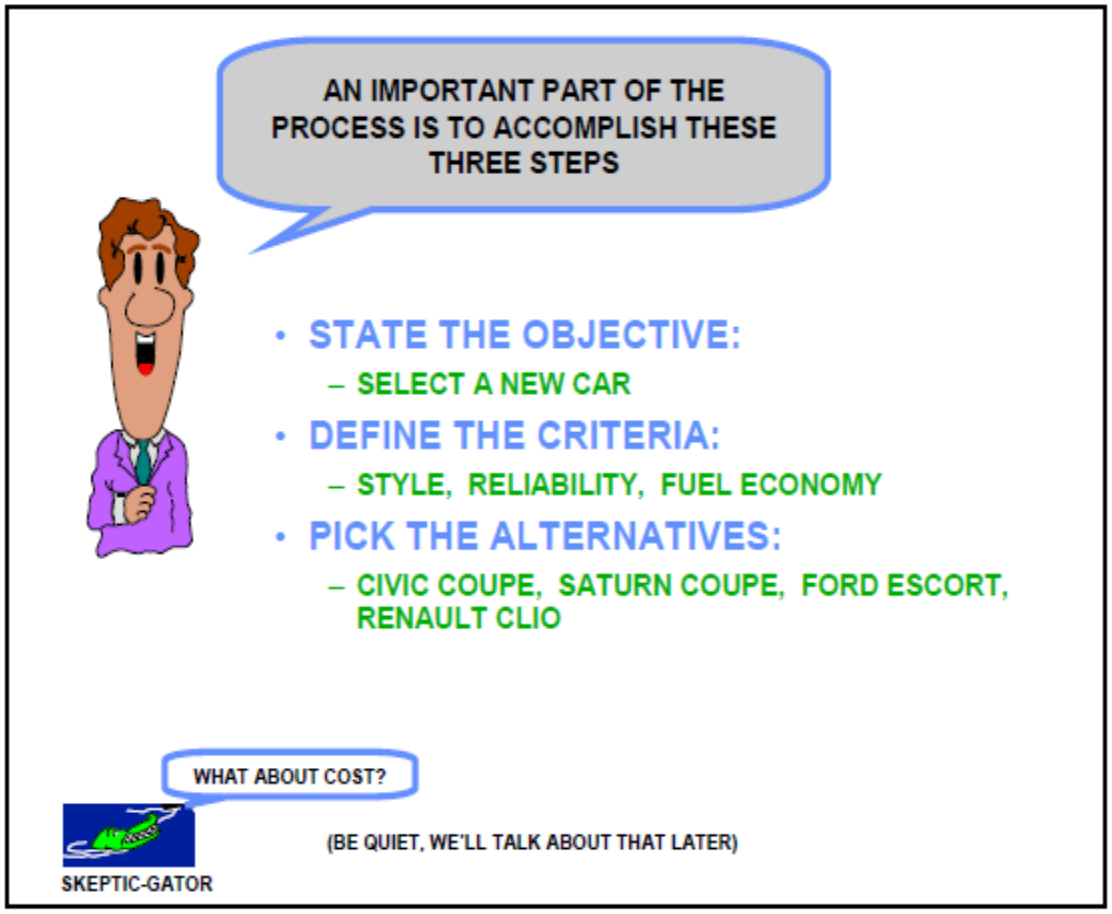
A [3] = (0.548 x 0.333) + (0.5 x 0.667)

A [3] = **0.515**

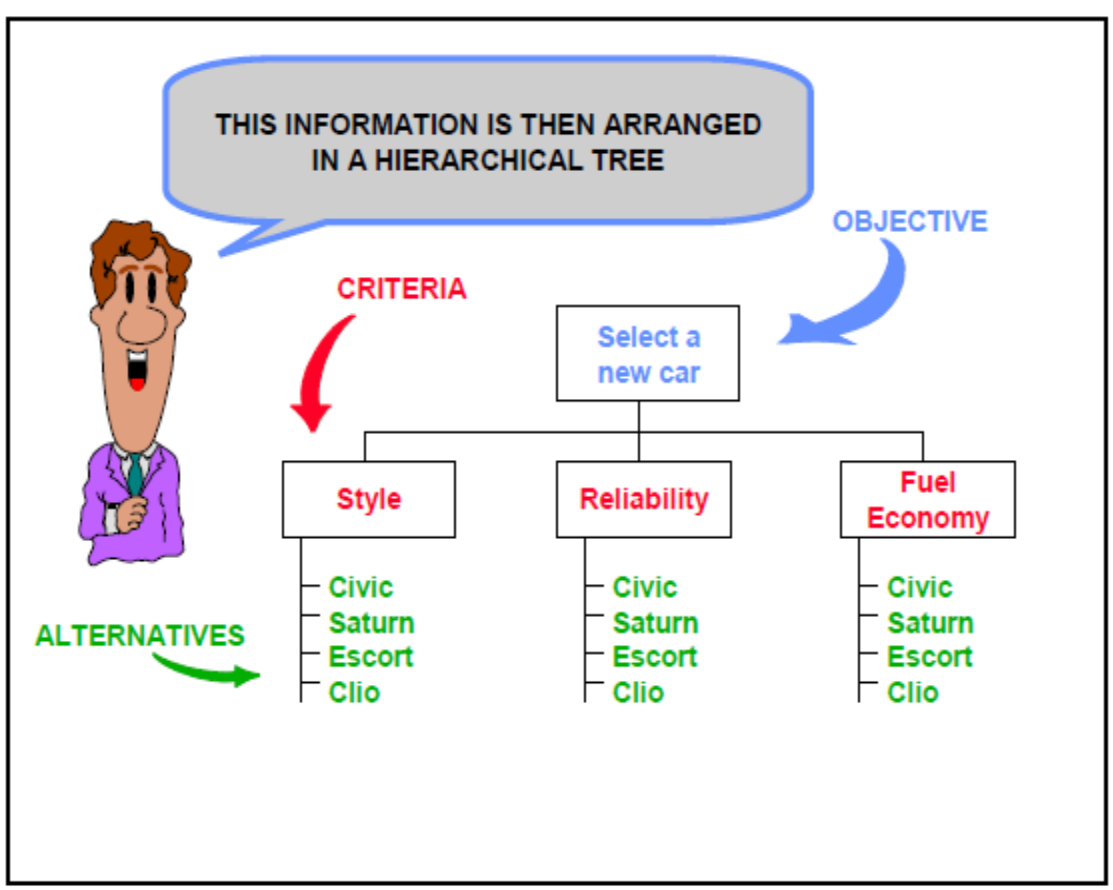
Here, OS/400 has the highest score, which indicates that OS/400 is the most suitable Operating System for the user. Since the requirement of the user is security, not cost, therefore Os/400 is the most suitable Operating System for user.

Hence we can safely conclude that AHP can be used for the selection of Operating systems.

Example of AHP: Car Selection



Example of AHP: Car Selection



Example of AHP: Car Selection

HERE'S ANOTHER WAY

Hmm, I think reliability is the most important followed by style and fuel economy is least importeant so I will make the following judgements ....

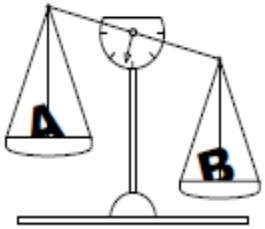
USING JUDGMENTS TO DETERMINE THE RANKING OF THE CRITERIA

1. RELIABILITY IS 2 TIMES AS IMPORTANT AS STYLE
2. STYLE IS 3 TIMES AS IMPORTANT AS FUEL ECONOMY
3. RELIABILITY IS 4 TIMES AS IMPORTANT AS FUEL ECONOMY

he's not very consistent here ... that's o.k.

Example of AHP: Car Selection

Pairwise Comparisons



USING PAIRWISE COMPARISONS, THE RELATIVE IMPORTANCE OF ONE CRITERION OVER ANOTHER CAN BE EXPRESSED

1 equal 3 moderate 5 strong 7 very strong 9 extreme

	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

Example of AHP: Car Selection

HERE'S OUR PAIRWISE MATRIX WITH THE NAMES

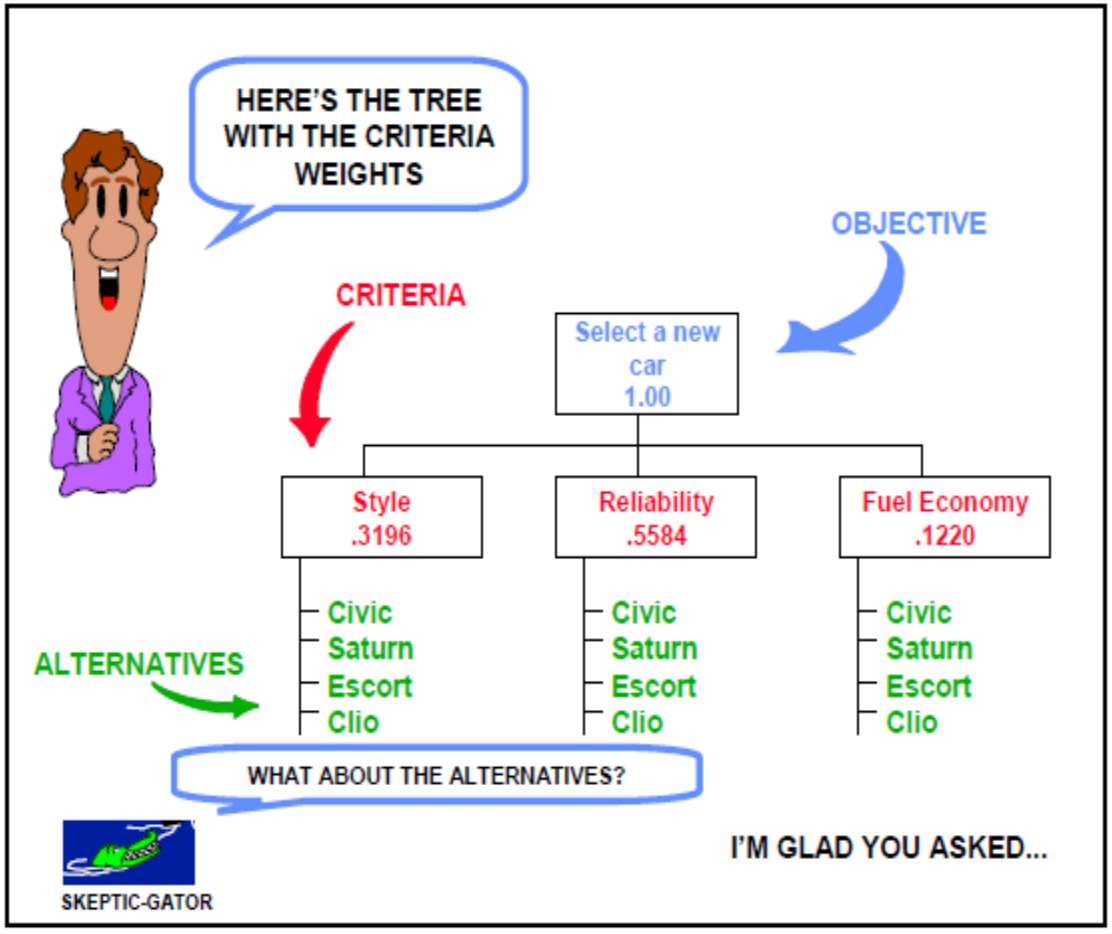
	STYLE	RELIABILITY	FUEL ECONOMY
STYLE	1/1	1/2	3/1
RELIABILITY	2/1	1/1	4/1
FUEL ECONOMY	1/3	1/4	1/1

AND THE COMPUTED EIGENVECTOR GIVES US THE RELATIVE RANKING OF OUR CRITERIA

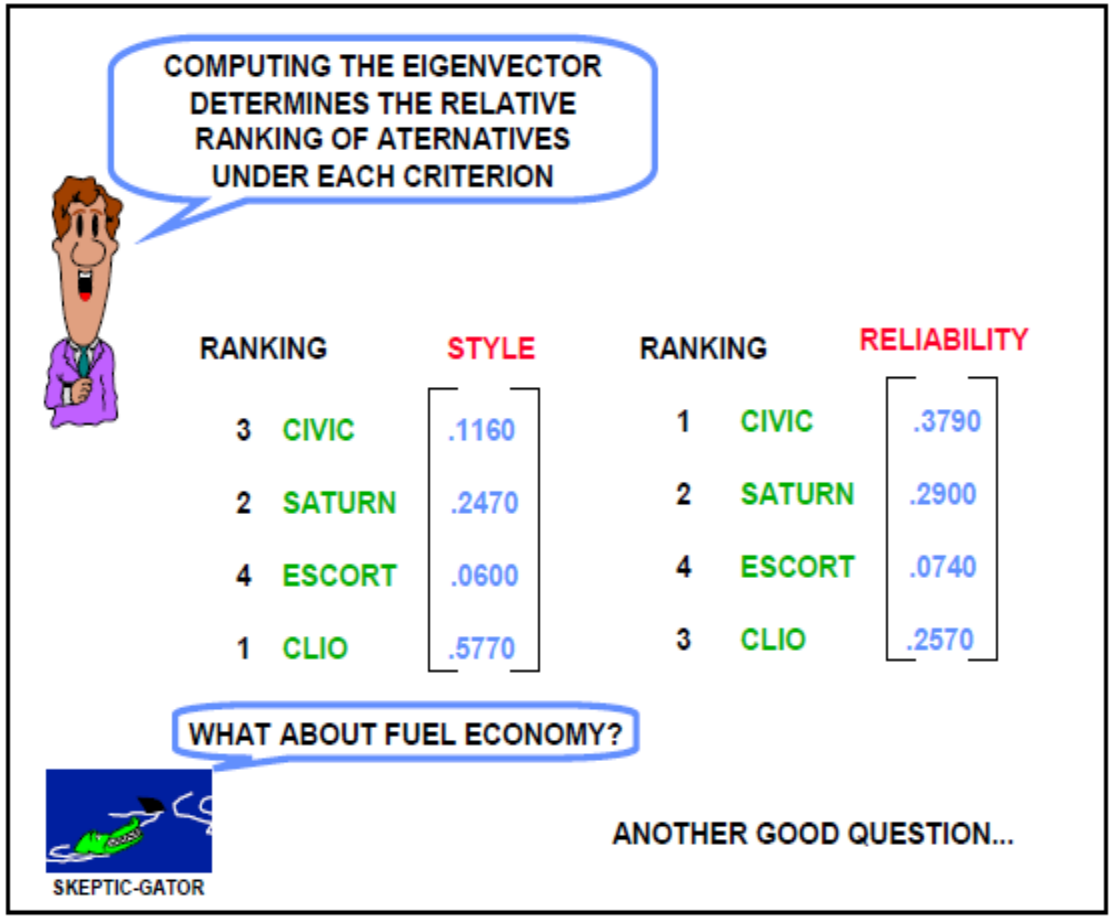
STYLE	0.3196	← THE SECOND MOST IMPORTANT CRITERION
RELIABILITY	0.5584	← THE MOST IMPORTANT CRITERION
FUEL ECONOMY	0.1220	← THE LEAST IMPORTANT CRITERION

NOW BACK TO THE HIEARCHICAL TREE...

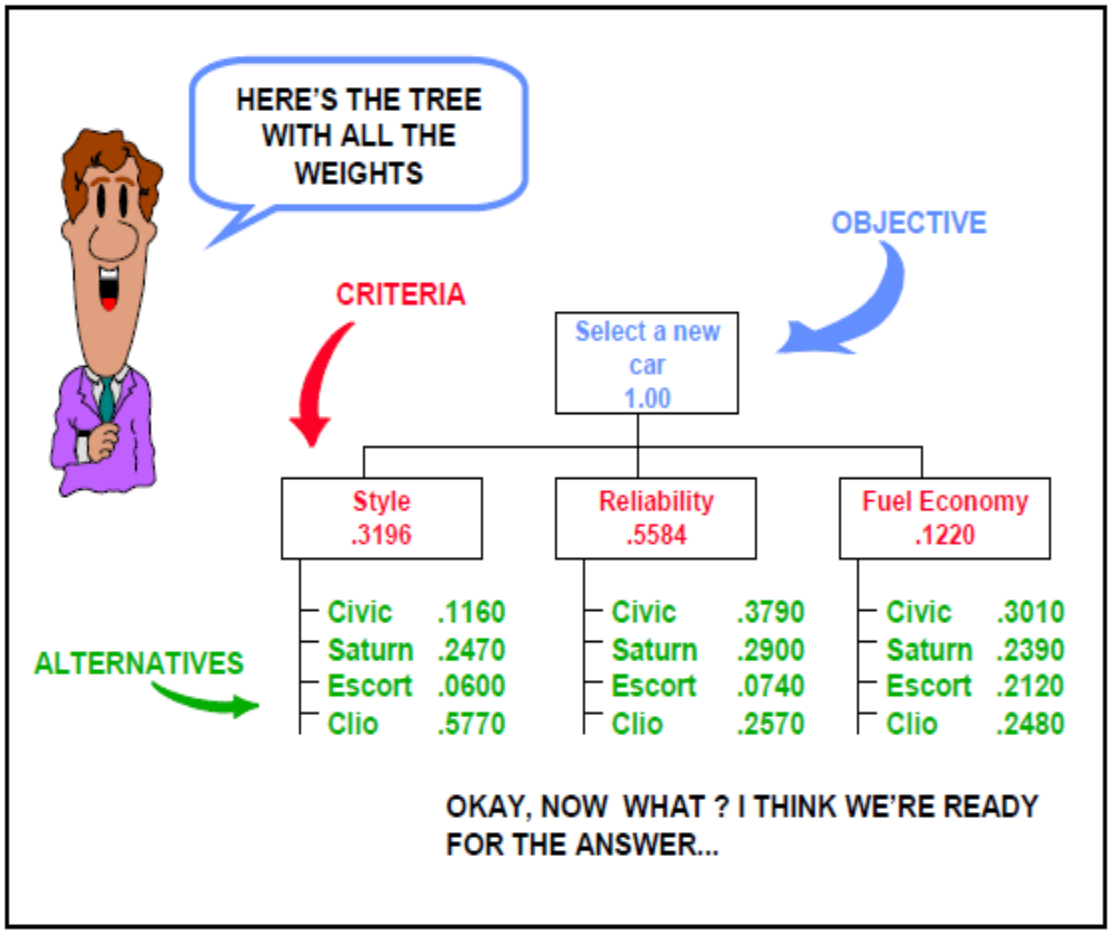
Example of AHP: Car Selection



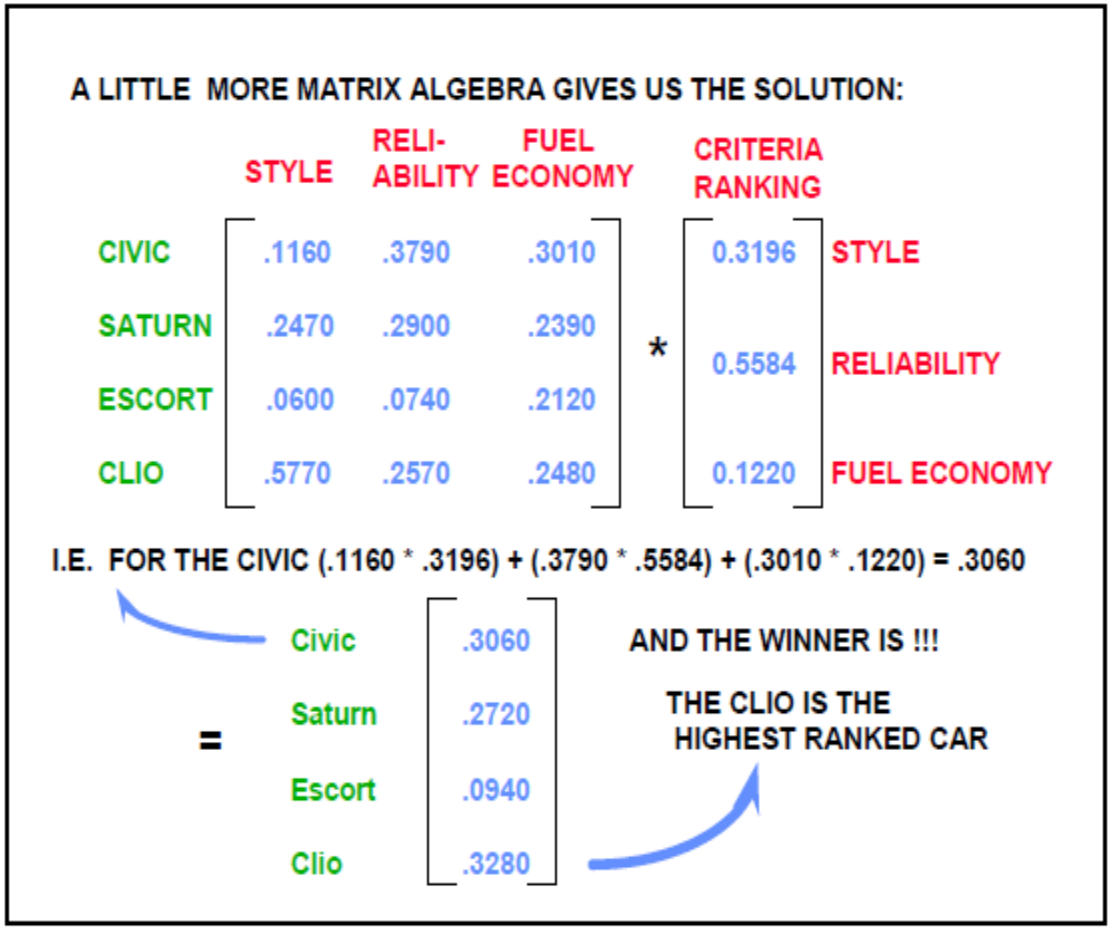
Example of AHP: Car Selection



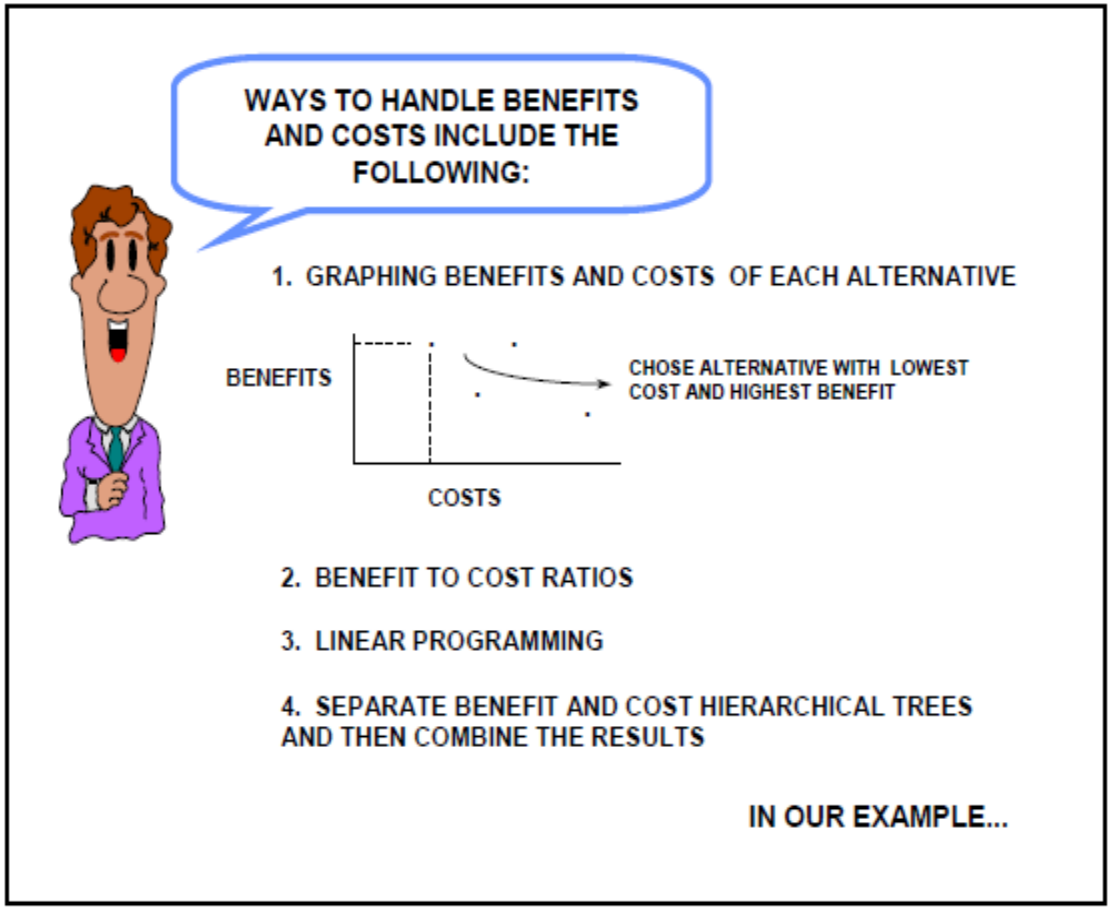
Example of AHP: Car Selection



Example of AHP: Car Selection



Example of AHP: Car Selection



Example of AHP: Car Selection

LET'S USE BENEFIT TO COST RATIOS

	COST $	NORMALIZED COSTS	BENEFIT - COST RATIOS
1. CLIO	18,000	.3333	.3280 / .3333 = .9840
2. CIVIC	12,000	.2222	.3060 / .2222 = 1.3771
3. SATURN	15,000	.2778	.2720 / .2778 = .9791
4. ESCORT	9,000	.1667	.0940 / .1667 = .5639
	54,000	1.0000	

(REMEMBER THE BENEFITS WERE DERIVED EARLIER FROM THE AHP)

AND...

THE CIVIC IS THE WINNER WITH THE HIGHEST BENEFIT TO COST RATIO

Applications of AHP

AHP can be used for a wide variety of applications:

- Strategic Planning
- Resource Allocation
- Source Selection
- Business/Public Policy
- Program Selection
- And much more....

Thank You